S ppor ing Informa ion

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In this document, we detail the projections performed by Bob's measurements and provide a detailed account of the states prepared during the experiment. We detail how Alice is able to transform her initial state $|3\rangle$ into the state $|I\rangle$, and subsequently transform $|F\rangle$ back into state $|3\rangle$. We describe our notation for probabilities that allow us to describe the "

A rotation through 2π radians introduces a sign change, such that there are two combined rotations through 2π, first on the $\{|3\rangle, |1\rangle\}$ level and then on the $\{|3\rangle, |2\rangle\}$ level. We have:

$$\begin{pmatrix} |1' \\ |3' \\ |2' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\hat{\ }_{IF}} \begin{pmatrix} |1\rangle \\ |3\rangle \\ |2\rangle \end{pmatrix} \qquad \textbf{[S14]}$$

The two rotations, each through 2π, have the combined effect of flipping the signs of the states $|1\rangle$ and $|2\rangle$ relative to state $|3\rangle$; specifically, we have:

$$|F\rangle = \hat{\ }_{IF}|I\rangle \qquad \textbf{[S15]}$$

Therefore, by applying these two rotations, Alice can map $|F\rangle \to |I\rangle \to |3\rangle$ and measure M_3 as per the main text.

C. Experimental Implementation of Nuclear Spin Readout. 1. Sample. We use a naturally occurring nitrogen vacancy (NV^-) center in high-purity (spin-bearing impurities controlled below 1 part per billion) type IIa diamond grown by chemical vapor dehe54.0597Tm470063T1.1128TL8.9692008.966338.38005515.2253T49.5857]TETwith or M_3) in the following readout, we probe all nuclear spin states within the [illegible] $= -1$ manifold in the same measurement run.

II. Analysis of the Leggett–

is not the case, we simply absorb the dynamics during intervals $_1 \ldots _2$ and $_2 \ldots _3$ into the definitions of the $_i$ measurements.

We then write $_1$, $_2$, and $_3$ as measurements along three directions $\hat{}_1$, $\hat{}_2$, and $\hat{}_3$, such that $\hat{}_i = \hat{\sigma} \cdot \hat{}_i$. For the purposes of evaluating the Leggett–Garg function, the important quantity is the inner product between the measurement directions. We define $\cos\theta_{ij} = \hat{}_i \cdot \hat{}_j$, and analysis shows that $\langle \quad _j \rangle_{(i,j)} = \cos\theta_{ij}$. In terms of this, the Leggett–Garg function becomes:

$$\langle K \rangle = \cos\theta_{12} + \cos\theta_{23} + \cos\theta_{13} \qquad \textbf{[S19]}$$

In the quantum case, we can pick three directions for $\hat{}_i$, such that $\theta_{12} = \theta_{23} = \theta_{13} = 120° = 2\pi/3$ radians. Because $\cos(2\pi/3) = -1/2$, this choice obtains $\langle K \rangle = -3/2$ when the quantum system is measured, violating the inequality.

2. Detectable disturbance during measurement. We define the detectable disturbance D as the difference in $\langle K \rangle$ induced by performing pairs of measurements, compared with performing all three measurements:

$$D = \langle \;_1 \;_2 \rangle_{(1,2)} - \langle \;_1 \;_2 \rangle_{(1,2,3)} + \langle \;_2 \;_3 \rangle_{(2,3)} - \langle \;_2 \;_3 \rangle_{(1,2,3)} + \langle \;_1 \;_3 \rangle_{(1,}$$

+1 with probability ● and $_2$ = −1 with probability 1 − ●. We have:

$$\langle \ _2 \ _3\rangle_{(\ _1, \ _2, \ _3)} = ● [P(\ _3 = +1| \ _2 = +1) - P(\ _3 = -1| \ _2 = +1)]$$
$$+ (1 - ●)[P(\ _3 = -1| \ _2 = -1) - P(\ _3 = +1| \ _2 = -1)]$$
$$= ● \cos^2(\theta_{23}/2) - ● \sin^2(\theta_{23}/2) + (1 - ●)\cos^2(\theta_{23}/2)$$
$$- (1 - ●)\sin^2(\theta_{23}/2) = \cos^2(\theta_{23}/2) - \sin^2(\theta_{23}/2) = \cos\theta_{23}$$

from which the influence of the $_1$ measurement represented by ● cancels, implying $D_{23} = 0$.

3. Evaluating **D**$_{13}$. We can see that $\langle \ _1 \ _3\rangle_{(\ _1, \ _3)}$ is insensitive to the state before $_1$, by substituting $_1 \to \ _2$ and following a similar argument as for $\langle \ _2 \ _3\rangle_{(\ _2, \ _3)}$ above. We show that $\langle \ _1 \ _3\rangle_{(\ _1, \ _2, \ _3)}$ is also insensitive to the initial state by assuming that the state before $_1$ measurement yields $_1$ = +1 with probability and $_1$ = −1 with probability 1 − . We have:

$$\langle \ _1 \ _3\rangle_{(\ _1, \ _2, \ _3)} = P(\ _2 = +1)\langle \ _1 \ _3| \ _2 = +1\rangle$$
$$+ P(\ _2 = -1)\langle \ _1 \ _3| \ _2 = -1\rangle$$
$$= \ P(\ _2 = +1| \ _1 = +1)[P(\ _3 = +1| \ _2 = +1)$$
$$- P(\ _3 = -1| \ _2 = +1)] + \ P(\ _2 = -1| \ _1 = +1)$$
$$\times[P(\ _3 = +1| \ _2 = -1) - P(\ _3 = -1| \ _2 = -1)]$$
$$+ \ (1 - \ )P(\ _2 = +1| \ _1 = -1)[P(\ _3 = +1| \ _2 = +1)$$
$$- P(\ _3 = -1| \ _2 = +1)] + (1 - \ )P(\ _2 = -1| \ _1 = -1)$$
$$\times[P(\ _3 = +1| \ _2 = -1) - P(\ _3 = -1| \ _2 = -1)]$$

This expression contains 16 terms, yielding:

$$\langle \ _1 \ _3\rangle_{(\ _1, \ _2, \ _3)} = \ \cos^2(\theta_{12}/2) \ \cos^2(\theta_{23}/2) - \sin^2(\theta_{23}/2)$$
$$+ \ \sin^2(\theta_{12}/2) \ \sin^2(\theta$$

This is outside the range $-1 \leq \langle K \rangle \leq 3$, providing an opportunity to detect an inconsistency with MR.

VI. Error Analysis of the Experimental Results

We find small deviations from the values expected from an ideal implementation. In the following, we give a brief description of the origin of these discrepancies and discuss their consequences on the macrorealist's possible conclusions.

) We find $P_{M} (B) < 1$ (i.e., there is not always a ball found in all the boxes). This is a consequence of a smaller than unity probability of correctly identifying the electronic $= 0$ state, resulting in an effective detection efficiency of $P \approx 90\%$. Although the macrorealist might conclude that there is not always an object hidden in the boxes, he still finds an unbiased initial state (within statistical uncertainty). Therefore, he cannot expect Alice to take advantage of this discrepancy. Based on his secret choice of M_1 or M_2 and the reduced probability of finding an object, he expects a maximum prob-

When using the complete register readout on $\ell = -1$ and $\ell = +1$, we have:

$$K^{\min}_{|\ell=\pm 1} = -1.1373 \quad \sigma^{\min}_{|\ell=\pm 1} = 0.0252 \quad (5.46\sigma \text{ violation})$$ **[S55]**

$$K^{\text{fair}}_{|\ell=\pm 1} = -1.1833 \quad \sigma^{\text{fair}}_{|\ell=\pm 1} = 0.0241 \quad (7.60\sigma \text{ violation})$$ **[S56]**

$$K^{\max}_{|\ell=\pm 1} = -1.2531 \quad \sigma^{\max}_{|\ell=\pm 1} = 0.0210 \quad (12.07\sigma \text{ violation})$$ **[S57]**

In the event, we found that the undetermined measurement outcomes do not give Bob suffi