# QUANTUM TIME

By

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**Abstract**

In quantum mechanics, time plays a role unlike any other observable. We find that measuring hether an event happened, and measuring hen an event happened are fundamentally different – the t o measurements do not correspond to compatible observables and interfere ith each other. We also propose a basic limitation on measurements of the arrival time of a free particle given by $1/\bar{E}_k$ here $\bar{E}_k$ is the particle's kinetic energy.

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## Dedication

In loving memory of my father, Peter Oppenheim (1942-1998) - my first physics teacher, ho encouraged my curiosity, patiently ans ered my questions, and patiently asked his o n. He ould have loved to flip through this thing, and I had al ays imagined giving him a copy.

# Chapter 1

# Introduction

## 1.1 Dual Measurements

One of the first lessons of quantum mechanics as that a property of a system does not correspond to an element of reality until it is measured. It makes no sense to talk about the position of a particle or the momentum of the particle, in and of itself. It is only hen e measure a physical quantity that e can actually say that a system possesses it. The particle does not have a position until its position is actually measured.

Ordinarily in quantum mechanics, one is interested in measuring properties of a system at a particular time $t$. One might ant to kno a particle's position, momentum, or spin, and the measurement of this quantity occurs at a certain time. For experiments at a fixed time, quantum mechanics provides us ith a useful formalism to describe reality.

the time becomes the observable one is trying to measure.

Classically, the time of an event can be made into an observable just like any other and this time can be measured in a variety of ays, all of hich give the same result. One can simply invert the equations of motion of the system to find the time that an event occurs [1], and then measure the values of the canonical variables (generalized coordinates and conjugate momenta). Since classically there is no uncertainty relation preventing the measurement of all the coordinates and conjugate momenta simultaneously, there is no limitation for finding the event's time. One could also continually monitor the system to determine the precise time hen the event occurred. Since one can make the interaction bet een the system and the measuring apparatus as small as one likes, this measurement need not disturb the evolution of the system. Finally, one can also couple a clock to the system in such a ay that the clock stops hen the event occurs. All these methods yield the same results, and ork to any desired accuracy.

Dual measurements, are quite common in modern laboratory experiments. In particle physics one often ants to kno the time that certain collision[garbled] u t ics o S e t euD hen

Pauli [8] as the first to u [garbled]monstrate that there as no operat[garbled] associated ith time. A time operat[garbled] must be conj

the time-of-arrival.

The interest in a quantum mechanical time operator stems in part from the troubling

It is space-time hich is the element of reality in general relativity.

These coordinates are, of course, subject to coordinate transformations, and in particular, the theory is invariant under reparametrization of the time coordinate. One consequence of this, is that if one tries to canonically quantize Einstein's theory of gravity in a closed system, one finds that the ave-function must satisfy the Wheeler-DeWitt equation

$$\mathcal{H}\Psi(g_{ab}, \pi_{ab}) = 0 \tag{1.1}$$

here the ave function depends on the 3-metric and conjugate momenta and $\mathcal{H}$ is exist many ambiguities in the role of time in quantum mechanics. Our hope is that a better understanding of time in the arena of quantum mechanics ill benefit and inform research in the field of quantum gravity. At the end of this thesis, e ill discuss some of the connections bet een the problem of time in quantum gravity and our research.

## 1.2 Differences Between Measurements of Space and Measurements of Time

Ever since Einstein's theory of special relativity, e have been encouraged to think of time and space on an equal footing. Ho ever, even classically, time and space are quite different as our common experience tells us. Objects move constantly for ard in time an a manner very different to the ay they move through space. Although e ill discuss in more detail the differences bet een quantum measurements of ordinary observables and measurements of time in Chapter 2, it may be instructive to roughly outline the differences bet een measurements of a particle's position at a fixed time, and the time a particle is found at a particular location.

In standard quantum mechanics, the probability that a particle is found at a given location $X$ at time $t$ is given by

$$P_t(X) = |\psi(X,t)|^2 \quad . \tag{1.2}$$

If e kno $\psi(x,0)$ for all $x$ then the system is completely described and e can easily compute this probability distribution at an instant of time. If e kno the Hamiltonian of the system, then using the Schrödinger equation e can also compute $\psi(x,t)$ at any time $t$. This probability distribution corresponds to results of a measurement of position at a particular time. Quantum mechanics gives a ell defined ans er to the question, " here is the particle at time $t$?"

Ho ever, it is also perfectly natural to ask "at hat time is the particle at a certain location." Here, quantum mechanics does not seem to provide an unambiguous ans er.

At first sight it seems that the probability distribution $P_x(T)$ to find the particle at a certain time at the location $x$ is simply $|\psi(x,T)|^2$, Ho ever, $|\psi(x,T)|^2$, does not represent a probability *in time*, since it is not normalized ith respect to $T$.

One might be tempted therefore, to consider the quantity

$$P_x(T) = \frac{|\psi(x,T)|^2}{\int |\psi(x,t')|^2 dt'} \tag{1.3}$$

This normalization depends on the particular state being measured, and can only be done if one kno s the state $\psi(x,t)$ at all times $t$ (infinitely far in the past and future). There are also states for hich the particle is never found at the position $x$, in hich case the expression above is undefined. Not- ithstanding this, one might argue that this quantity gives one a relative probability that the particle is found at the location $x$ at time $T$ (if the measurement is made at that time $T$), as opposed to another time $T'$ (if the measurement is made at time $T'$).

Ho ever, the expression above certainly does not yield the probability *in time* to detect the particle. One reason for this failure is that a particle may be detected at a location $X$ at many different times $t$ (e.g. I can be found in my office at many different times in the day). On the other hand, if at time $t$ a particle is detected at location $X$, then e can say ith certainty that at the same time $t$, the particle as not at any other location $X'$ (e.g. at nine a.m. I am in bed, and therefore, I cannot also be in my office). Equation (1.3) does not give a proper probability distribution as the various outcomes are not disjoint. $P_x(T)$ is not a probability distribution in time in the sense usually reserved for probability distributions in quantum mechanics. $P_x(T)$ is very different from $P_t(X)$ and has different properties (as e ill see in the next chapter).

This leads us to consider the time of first arrival of a particle, since a particle can only arrive once to a particular location. In order to measure the arrival time one cannot use expression (1.3) since one needs to detect the particle at time $t_A$, and also kno that the particle as not there at any previous time. In other ords, one must continuously monitor the location $x_A$ in order to find out hen the particle arrives. Ho ever, this continuous measurement procedure has it's o n difficulty, and also emphasizes the problem ith the previous probability distribution. Namely, that the probability to find a particle at $t = T$ is generally *not* independent of the probability to find the particle at some other time $t = T'$. i.e.. if $\mathbf{\Pi}_{x_A}$ is the projector onto the position $x_A$, then in the Heisenberg representation [3]

$$[\mathbf{\Pi}_{x_A}(t), \mathbf{\Pi}_{x_A}(t')] \neq 0. \tag{1.4}$$

Measurements made at different times disturb each other. We ill see in Section 2.2 that this is one of the properties of ordinary measurements hich measurements in time violate. Measurements made at different times do not commute. Therefore the probability distribution obtained from this measurement procedure, although ell defined, does not give a probability distribution *in time*.

Von Neumann measurements [4] happen *at a certain time*. One measures the particle's position at time $t$. Even a continuous measurement at a particular location is a series of measurements at a certain time. Each instant that the Geiger counter doesn't click, it is measuring the fact that a particle has not entered it. Furthermore, operators hich are used to measure the time-of-arrival to the location $x_A$, are not measured at $x_A$, but rather at an instant in time. In quantum mechanics, measurements made at different times can disturb each other, hich can make measurements of the time of an event problematic. The probability of detecting a particle at a certain location at time $t$ is not independent of detecting the particle at some other time $t'$.

## 1.3 Inaccuracies and Uncertainties

The measurement of an observable corresponding to a self-adjoint operator can be as accurate as one ishes. This is true despite any uncertainty relations hich govern various sets of observables. The position, or momentum of a particle (but not both) can be measured to any desired precision. Consider t o observables **A** and **B** hich do not evolve in time, and hose commutator is $i$ (in units here $\hbar = 1$). Imagine that e have an ensemble of identical systems prepared in some initial state. On half the ensemble, e can measure **A**, and on the other half, e can measure **B**. Each individual measurement can be as accurate as e ish. An extraordinary experimentalist can reduce the inaccuracies in the measurement to almost zero, and can get a particular value for each measurement. The experimentalist may have a dial on her device hich ill point to the value of $A$ after the measurement. She ill have to make sure that initially the pointer on her dial points almost exactly to zero, and then after each run of her experiment, she measures the position of the dial very accurately to determine the value of $A$.

If e then plot all of the measurements of **A** and all of the measurements of **B**, e ill find a distribution of measurements hich have a natural idth of $\Delta A$ and $\Delta B$ respectively. One then finds that no matter hat initial state e choose, $\Delta A \Delta B > 1$. There is an *uncertainty* relation bet een the distributions of $A$ and $B$, but there are no theoretical limitations on the accuracy of each individual measurement of **A** or **B**.

The experimentalist does not have to make her measurements totally precise. She could, for example, start off the experiment ith her dial in a state here the initial position of the needle is *uncertain*. An uncertainty in the initial pointer position ill result in her measurement being *inaccurate*. When she measures the final position of her

of the probability current to measure the time at hich a particle arrives to a certain location. The discussion suggests that the difference bet een time and other observables is not merely formal.

The central result of the thesis is contained in Chapter 3 here e discuss the problem of the time-of-arrival of a particle to a particular location. It is argued that the time-of-arrival cannot be precisely defined and measured in quantum mechanics. By constructing relationship bet een these modified operators, and the direct measurements discussed in Chapters 2 and 3, and argue that a measurement of the time-of-arrival operator does not correspond to these continuous measurements. Unlike the classical case, in quantum mechanics the result of a measurement of the time-of-arrival operator may have nothing to do ith the time-of-arrival to $x = x_A$.

There has been rene ed interest in time-of-arrival operators follo ing the suggestion by Grot, Rovelli, and Tate, that one can modify the lo momentum behavior of the operator slightly in such a ay as to make it self-adjoint [9]. We sho that such a modification results in the difficulty that the eigenstates are drastically altered. In an eigenstate of the modified time-of-arrival operator, the particle, at the predicted time-of-arrival, is found far a ay from the point of arrival ith probability 1/2.

The bound of $1/\bar{E}_k$ on the accuracy of time-of-arrival measurements is based on calculations done using numerous measurement models corresponding to specific Hamiltonians, as ell as more general considerations. Ho ever, because the limitation is based on dynamical considerations and not kinematic ones, a formal proof of the limitation may not exist. For example, a proof of the Heisenberg uncertainty relation relies only on the properties of specific operators, hile our inaccuracy relation is a statement not about operators, but about measurements (and therefore, involves the dynamical considerations of the actual measurement). Perhaps by making certain restrictive assumptions about the Hamiltonian one might be able to construct a formal proof. Such a proof ould have to take into account the measurement model hich ill be discussed in Section 3.3.3 in hich e sho that if one has prior information about the avefunction, and if the avefunction is almost an eigenstate of energy (i.e. its time of arrival is completely uncertain), then one can measure the time of arrival to an accuracy better than $1/\bar{E}_k$. One therefore expects that a formal proof ill not only have to involve making assumptions about the interaction Hamiltonian, but also the initial state of the ave function. The existence of a formal proof for our inaccuracy limitation remains an interesting open question.

While e kno of no formal proof for the inaccuracy limitation for time-of-arrival, one can make more general statements about measurements of "traversal time". In Chapter 5 e consider the problem of a free particle hich traverses a distance $L$ and argue that a violation of the above limitation for the traversal-time implies a violation of the Heisenberg uncertainty relation for $x$ and $p$. This result does not depend on the details of the model being used in the measuring process. Measurements of traversal-time are dual to measurements of traversal distance, and it can be sho n that one can measure the distance a particle travels to any desired precision. This chapter also contains a further discussion on the difference bet een hat e call "inaccuracy" limitations, hich constrain the precision ith hich individual measurements are performed, and "uncertainties" hich are kinematic quantities hich relate to the spread in measurements on ensembles.

Chapter 6 contains hat may be our most interesting result. In it, e examine hether one can determine the temporal ordering of events. We find that one cannot measure hether one event occurred in the future or past of another event to arbitrary accuracy. The minimum inaccuracy for measuring hether a particle arrives to a given location before or after another particle is given by $1/\bar{E}$ here $\bar{E}$ is the total kinetic energy of the t o particles. We discuss the relationship bet een this type of measurement, and coincident counters, as ell as Heisenberg's microscope. We sho that in general one cannot prepare a t o particle state here the t o particles al ays arrive ithin a time of $1/\bar{E}$ of each other. This has interesting consequences for determining the metric properties of a space-time.

In this thesis e ill ork in units here $\hbar = c = 1$