

# MATH0018 Functional Analysis

<i>Year:</i>	2024{2025
<i>Code:</i>	MATH0018
<i>Level:</i>	6 (UG)
<i>Normal student group(s):</i>	UG: Year 3 Mathematics degrees
<i>Value:</i>	15 credits (= 7.5 ECTS credits)
<i>Term:</i>	2
<i>Assessment:</i>	90% examination, 10% coursework
<i>Normal Pre-requisites:</i>	MATH0051
<i>Lecturer:</i>	Dr M Karpukhin

## *Course Description and Objectives*

Elementary analysis mostly studies real-valued functions on the real line  $\mathbb{R}$  or on  $n$ -dimensional space  $\mathbb{R}^n$ . Functional analysis, by contrast, shifts the point of view: we collect all the functions of a given class (for instance, all bounded continuous functions) into a *space of functions*, and we study that space (and operations on it) as an object in its own right. Since spaces of functions are nearly always infinite-dimensional, we are led to study analysis on infinite-dimensional vector spaces, of which the most important cases are Banach spaces and Hilbert spaces. This course provides an introduction to the basic concepts of functional analysis. These concepts

Normed linear spaces and Banach spaces. Examples: Sequence spaces  $\ell^p$  ( $1 \leq p < \infty$ ) and  $c_0$ ; spaces  $C(X)$  of bounded continuous functions. Proofs of completeness of these spaces. Special properties of finite-dimensional normed linear spaces.

Hilbert spaces. Basic properties. Orthogonal systems and the orthogonalization process. Representation of linear functionals on Hilbert space.

Zorn's lemma and the Hahn-Banach theorem.

Linear functionals and duality. Dual of  $\ell^p$  is  $\ell^q$ . Second dual and reflexive spaces.

Baire category theorem. Uniform boundedness theorem, open mapping theorem, closed graph theorem.

Weak and weak- $^*$  topologies. Weak- $^*$  compactness of the unit ball in the dual space. Compact operators.