probabilities), although the logistic function can be applied to obtain the log odds. In this paper we consider three potential candidates: Cramér's ϕ, adjusted C and Bayesian dependency.

Given this range of potential measures for effect size, two questions arise.

ϕ has a value of 0 when there is no association between the two values, that is, when the probability of selecting a value of *A* is constant for any value of *B*.

2. **█n   n   nc**   In Appendix 1 we prove that Cramér's $\phi(A, B)$ ϕ measures the linear interpolation from flat to identity matrix (see Figure 1 for the idea). We can therefore refer to ϕ as the best estimate of the population **n   n   n   o   ◄ y** *idp* $\equiv$ *p*(*a* $\leftrightarrow$ *b*). This intercorrelation is robust, i.e. it 'gracefully decays' the further it deviates from this ideal. ϕ is

$$dp_R(A, b) \equiv - \sum_{i=}^{k} {}_R\ a_i\ b$$

We note the following.

1. Relative dependency $dp_R(B, A)$ is linear with x when $p(a)$ is even.

2. Measures are ordered in size: $C_{adj} > dp_R(B, A) > \phi > dp_R(A, B)$.

3. $dp_R(A, B)$ (and therefore ϕ) converges to $dp_R(B, A)$ as $p(b)$ becomes more even (tends to $1/k$).

Whereas dp_R measures the distance on the first parameter from the prior (and is thus directional when a prior skew is applied to one variable only), ϕ is based on the root mean square distance of both variables. C_{adj} appears to behave rather differently to ϕ, as the right hand graph in Figure 2 shows. Given that the only other bi-directional measure, ϕ, measures the interdependence of A and B, there appears to be little advantage in adopting the less conservative C_{adj}.

Finally, for $k = 2$ the following equation also holds:

4. $\phi^2 = dp_R(A, B) \times dp_R(B, A)$.

We find an equality between a classical Bayesian approach to dependency and a stochastic approach based on Pearson's χ^2 for one degree of freedom. The proof is given in Appendix 3.

This raises the following question: what does 'directionality' mean here?

Note that dp_R

6. A worked example

Figure 3 provides a demonstration of plotting confidence intervals on ϕ

Appendix 1. The best estimate of population interdependent probability is Cramér's ϕ

Cramér's ϕ is not merely a 'measure of association'. It repres

Appendix 2. Deriving 2 × 2 rule dependency

For a 2×2 table with a single degree of freedom, the following axioms hold.

A1. $p(a_2) = p(\neg a_1) = 1 - p(a_1); \; p(a_2 \mid b_i) = p(\neg a_1 \mid b_i) = 1 - p(a_1 \mid b_i),$

A2. $p(a_1 \mid b_i) - p(a_1) = p(a_2) - p(a_2 \mid b_i),$

A3. $[p(a_1 \mid b_i) < p(a_1)] \leftrightarrow [p(a_2) < p(a_2 \mid b_i)].$

A1 is a consequence of the Boolean definition of A, A2 can be demonstrated using Bayes' Theorem and A3 is a consequence of A2. A2 further entails that row sums are equal, i.e. $dp_R(a_1, B) = dp_R(a_2, B)$.

Equation (5) may therefore be simplified as follows

$$dp_R(A, B) \equiv \frac{1}{k} \sum_{i=1}^{k} \sum_{j=1}^{k} dp_R(a_i, b_j) \times p(b_j) = \sum_{j=1}^{k} dp_R(a_1, b_j) \times p(b_j)$$

$$= \frac{p(a_1 \mid b_1) - p(a_1)}{1 - p(a_1)} \times p(b_1) + \frac{p(a_1) - p(a_1 \mid b_2)}{p(a_1)} \times p(b_2).$$

Applying Bayes' Theorem $(p(a_1 \mid b_2) \equiv p(b_2 \mid a_1) \times p(a_1) / p(b_2))$ and axiom A1:

$$dp_R(A, B) = \frac{p(a_1 \mid b_1) p(b_1)}{1 - p(a_1)} - \frac{p(a_1) p(b_1)}{1 - p(a_1)} + \frac{p(a_1 \mid b_1) p(b_1)}{p(a_1)} - p(b_1).$$

The first and third terms then simplify to $[p(a_1 \mid b_1)p(b_1)] / [(1 - p(a_1))p(a_1)]$, so

$$dp_R(A, B) = \frac{p(a_1 \mid b_1) p(b_1) - p(a_1)^2 p(b_1) - (1 - p(a_1)) p(a_1) p(b_1)}{(1 - p(a_1)) p(a_1)}$$

$$= \frac{[p(a_1 \mid b_1) - p(a_1)] p(b_1)}{[p(a_1 \mid b_1) - p(a_1)] p(a_1)}.$$

Appendix 3. For a 2 × 2 table, $\phi^2 \equiv dp_R(A, B) \times dp_R(B, A)$

The proof is in three stages.

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STAGE 2. Converting to a+b+c+d notation.

The 2×2 χ^2 statistic, and thus ϕ, may be represented simply in terms of four frequencies in the table, a, b, c and d (note roman font to distinguish from a, a_1, etc). The table is labelled thus, and $N \equiv$ a+b+c+d:

	b_1	b_2	Σ
a			